Math 1010 March 6, 2019 Homework 15

1) Sketch the following parabola clearly labeling the vertex, roots, and axis of symetry.

$f(x) = x^2 + 6x - 7$

The vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

The axis of symetry: The vertical line $x = \frac{-b}{2a}$

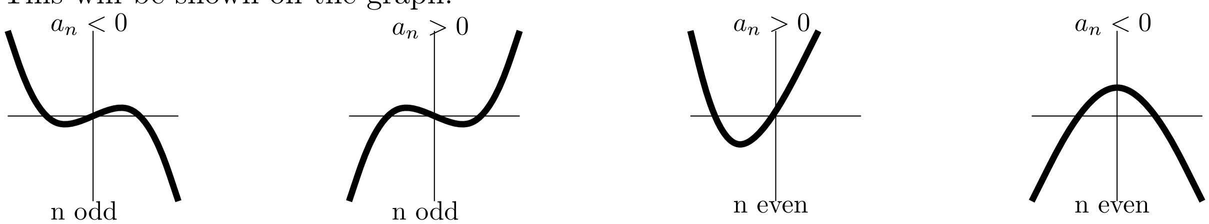
The roots are the solutions to $0 = ax^2 + bx + c$

2) A farmer want to create a rectangular field with one side along a straight stream using 1100 feet of fencing, but there will be fencing only on 3 sides and not on the side bordered by the stream. What dimensions will maximize the area of the field?

HINT: Use the force! Use Algebra!
Draw a picture. The area of the field is $A = L \cdot W$. Let x be the width of the field W, solve for L in terms of x, then express area A as a function of x. Draw a sketch of the curve $f(x) = A$ where A is a function of x, then find the vertex of this parabola.

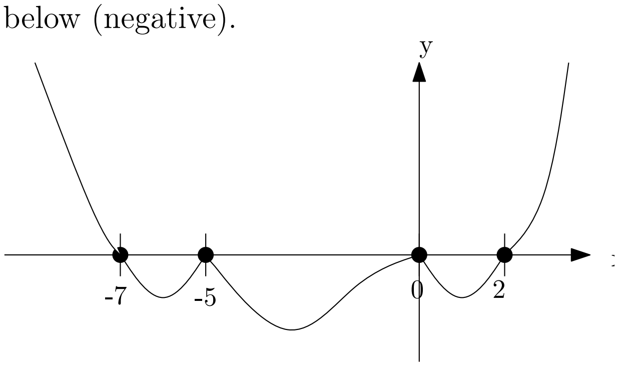
For each polynomial in factored form, show:
a) the leading term
b) the zeros on the x-axis
c) the general shape of the polynomial

EXAMPLE: $y = x^2(x-2)(x+5)^2(x+7)$
a) The leading term is x^6. The behavior as $x \to \pm\infty$ is found by observing the leading term $a_n x^n$. This will be shown on the graph.



In this case for x^6, n is even and the leading coefficient is 1 which is positive, so the gross shape will be like the 3d graph from the left.
b) The zeros are $0, 2, -5, -7$ found by setting each term equal to zero and solving for x. These will be shown on the sketch of the polynomial, here on a real number line.

c) The graph is above or below the axis between successive zeros, so one test point between zeros is enough to find out if it is above (positive) or below (negative).



Or, one can note that for roots of even multiplicity the graph will touch them without passing through, and the graph will pass through roots of odd multiplicity. For this polynomial, roots 0 and 5 are of even multiplicity (two) and just touch, while roots -5 and -7 have odd multiplicity and will pass through. One can start at the left noting the gross shape of the graph and complete the sketch touching or passing through each root in order. Note that the actual graph of the polynomial will look quite different. Here we are observing the shape with respect to roots, where the polynomial is positive and negative, and the bahavior of the polynomials as x goes to plus or minus infinity.

1. $y = (x-1)(x+3)(x-5)$

2. $y = (x+6)^3(x-1)^4(x+2)^5$

3. $y = x(x-1)(x+2)(x-3)(x+4)(x-5)(x+6)$

4. $y = x^3(x+7)(x-3)^2$