Math 1010 Spring 2017 Sample Test 2 KEY

1. What is the domain of $f(x) = \frac{x^2+1}{\sqrt{2x-5}}$?
   The denominator cannot be zero, and the radicand (thing inside the radical) must not be negative.
   Thus

$$\begin{aligned} 2x-5 &> 0 \\ 2x &> 5 \\ x &> \frac{5}{2} \end{aligned}$$

2. Find the equation of the line through the points (2, 3) and (-6, 9).
   $m = \frac{y_2-y_1}{x_2-x_1} = \frac{9-3}{-6-2} = \frac{6}{-8} = -\frac{3}{4}$

$$\begin{aligned} y &= m(x-x_1)+y_1 \\ y &= -\frac{3}{4}(x-2)+3 \\ y &= -\frac{3}{4}x+\frac{3}{2}+3 \\ y &= -\frac{3}{4}x+\frac{9}{2} \end{aligned}$$

3. $f(x) = 3x+2 \qquad g(x) = x^2$. Find $f(g(4))$.
   $g(4) = 4^2 = 16$
   $f(g(4)) = f(16) = 3 \cdot 16 + 2 = 48 + 2 = 50$

4. Find the vertex of the parabola $f(x) = x^2 - 6x - 4$
   HINT: The vertex of parabola $f(x) = ax^2+bx+c$ is the point $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
   $\frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$
   $f\left(\frac{-b}{2a}\right) = f(3) = 3^2 - 6(3) - 4 = 9 - 18 - 4 = -13$
   The vertex is the point $(3, -13)$

5. $p(x) = 2x^3 - x^2 + 3x + 1$ Find $p(-2)$ using synthetic division.

-2	2	-1	3	1
	⇓	-4	10	-26
	2	-5	13	-25

   $p(-2) = -25$

6. Find a polynomial having roots 2, 3, and -7.
   $(x-2)(x-3)(x-[-7]) = (x-2)(x-3)(x+7)$

7. Find the roots: $(x-3)(x+6)x = 0$
   HINT: Set each factor equal to zero and solve each for x.

$$x-3=0 \qquad x+6=0 \qquad x=0$$
$$x=3 \qquad x=-6$$

8. Compute: $\ln 32.89 = 3.493$

9. Compute: $\log 2.16 \times 10^{5000}$ HINT:$\log XY = \log X + \log Y$
   $\log 2.16 \times 10^{5000} = \log 2.16 + \log 10^{5000} = .3345 + 5000 = 5000.3345$

10. Compute:$\log_2 3.4$ HINT:$\log_b x = \frac{\log x}{\log b}$
$\log_2 3.4 = \frac{\log 3.4}{\log 2} = \frac{.5315}{.3010} = 1.7658$

11. Solve for x:$\log_3 4x^2 = \log_3 3x + 5$

$$\begin{aligned} \log_3 4x^2 &= \log_3 3x + 5 \\ \log_3 4x^2 - \log_3 3x &= 5 \\ \log_3 \frac{4x^2}{3x} &= 5 \\ \log_3 \frac{4}{3}x &= 5 \\ \frac{4}{3}x &= 3^5 = 243 \\ x &= \frac{3}{4} \cdot 243 = 182.25 \end{aligned}$$

12. Solve for x: $2^{3x-5} = 16$ HINT: $b^x = b^y \Rightarrow x = y$; Make the bases the same.

$$\begin{aligned} 2^{3x-5} &= 16 \\ 2^{3x-5} &= 2^4 \\ 3x - 5 &= 4 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

13. Solve for x: $\log_x 49 = 2$ HINT: Use the definition $\log_b a = c \longleftrightarrow b^c = a$
Then $x^2 = 49$, and $x = 7$. Note that the base must be positive, so -7 is not a solution.

14. Solve for x: $\log_2 x + \log_2 2 = 3$

$$\begin{aligned} \log_2 2x &= 3 \\ 2x &= 2^3 = 8 \\ x &= 4 \end{aligned}$$

15. Solve for x: $10^{2x} = 7$ HINT: log both sides using the common base 10 log.

$$\begin{aligned} 10^{2x} &= 7 \\ \log 10^{2x} &= \log 7 \\ 2x &= .8451 \\ x &= .4226 \end{aligned}$$

16. Solve for x: $\ln x + 2 \cdot \ln 5 = \ln 30$

$$\ln x + 2 \cdot \ln 5 = \ln 30$$

$$\begin{aligned} \ln x + 3.2189 &= 3.4012 \\ \ln x &= 0.1823 \\ e^{\ln x} &= e^{0.1823} \\ x &= 1.2 \end{aligned}$$

17. Suppose that \$1000 is invested at 5% APR compounded continuously. How log till the investment reaches a value of \$2000?
We use the equation $A = Pe^{rt}$ where A=2000, P=1000, and r=5%=.05. Then, solve for t.

$$\begin{aligned} 2000 &= 1000e^{.05t} \\ 2 &= e^{.05t} \\ \ln 2 &= \ln e^{.05t} \\ .6931 &= .05t \\ 13.862 &= t \end{aligned}$$

Hence, 13.862 years.

A general population growth problem.

$$p = p_o e^{rt}$$

p = population after t years $\quad$ p_o= population when t = 0
r = annual growth rate AS A DECIMAL $\quad$ t = years

The population of India is 1.148 billion in 2008 with an annual growth rate of 1.38%.

18. Assuming India's growth rate remains constant, what will the population be in 10 years?
$p = 1.148e^{.0138(10)} = 1.148(1.148) = 1.318$ billion

19. Assuming India's growth rate remains constant, in how many more years would India's population reach 1000 billion?

$$\begin{aligned} 1000 &= 1.148e^{.0138t} \\ 871.08 &= e^{.0138t} \\ \ln 871.08 &= \ln e^{.0138t} \\ 6.7697 &= .0138t \\ 484.04 &= t \end{aligned}$$

In 484.04 years

A decay problem.

$$y = y_o e^{kt}$$

y = amount after t years $\quad$ y_o = amount when t = 0
k = decay constant = $\frac{\ln .5}{half-life}$ $\quad$ t = years

Suppose that the half-life of an isotope is 8.6 years.

20. Find k. $\quad k = \frac{\ln .5}{8.6} = \frac{-.6931}{8.6} = -.0806$

21. How long till 23.2 grams of this isotop decays to .07 grams?

$$\begin{aligned} .07 &= 23.2e^{-.0806t} \\ .00302 &= e^{-.0806t} \\ \ln .00302 &= \ln e^{-.0806t} \\ -5.8025 &= -.0806t \\ 71.99 &= t \end{aligned}$$

In 72.35 years.