Math 2002 Spring 2017 Sample Final Exam

Show your work completely. Use sentences to explain your work as needed.

1. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for
$$z = \ln(x^5 + y^2)$$

2. Find the first partial derivatives of the function
$$f(a, b, c) = ab^3 - a^2c^4 + \frac{b}{c}$$
with respect to each independent variable.

3. Find all second order partial derivatives of the function
$$p(x, y) = x^2y^3 - y^3 + x^2y$$

4. Find the value of $\frac{\partial y}{\partial z}$ at the point $(4, 0, 2)$ if the equation
$$x^3z + y\ln x - x^2 + 4yz^2 = 112$$
defines y as a function of the two independent variables x and z, and the partial derivatives exist.

5. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ given the equations
$$\begin{aligned} w &= ab - bc + a^2b^3c^4 \\ a &= \ln(xy) \\ b &= \tan x + y^2 \\ c &= x^2y^2 \end{aligned}$$

6. The voltage V is a Jupiter space probe circuit, that satisfies the law $V = IR$, is slowly dropping as the battery loses solar cell input. At the same time, the restance R is decreasing as the probe passes into the shadow of Titan. Use the equation
$$\frac{dV}{dt} = \frac{\partial V}{\partial I}\frac{dI}{dt} + \frac{\partial V}{\partial R}\frac{dR}{dt}$$
to find how the current is changing (with respect to time) at the instant when $R = 80$ ohms, $I = 4.8$ amps, $\frac{dR}{dt} = -1.1$ohm/sec, and $\frac{dV}{dt} = -0.15$ volt/sec.