Math 2002 Spring 2017 Sample Test 2

1. Find the sum of the geometric series
$\frac{1}{3} - \frac{2}{15} + \frac{4}{75} - \frac{8}{375} + - \cdots$

2. Show that the series converges using the integral test: $\sum_{n=1}^{\infty} \frac{1}{n^3}$

3. Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(7x-4)^n}{\sqrt[3]{n}}$

4. Find the first four terms of the Taylor series about $x = 1$ for
$f(x) = \frac{1}{\sqrt{x}}$

5. Consider the the Taylor series for $\sin x$ about $x = 0$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \cdots$
How many terms are necessary to approximate $\sin 0.7$ accurate to within 0.001 ? Show this sufficiently accurate Taylor polynomial.

6. Use power series to solve the initial value problem.
$y' + y = 0, \quad y(0) = -1.$

7. Mark each of the following with C for convergent or D for divergent.
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

$\sum_{n=1}^{\infty} \frac{100^n}{n!}$

$\sum_{n=1}^{\infty} \frac{n^7}{7^n}$

$\sum_{n=1}^{\infty} \frac{n^n}{10^n}$

8. Find the exact value for each.
$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{n}$

$\lim_{n\to\infty} \sqrt[n]{n!}$

$\sum_{n=1}^{\infty} (2x-7)^n$ if $x = \pi$