Math 2002 April 10, 2017 Sample Test 4 (chapters 10-11)

Consider the vectors $A =< 5, 3, -2 >$, $B =< 2, -1, 0 >$.

1) Find $|A|$

2) Find $A \bullet B$

3) Find $A \times B$

4) Find $Proj_B A$

5) Find the angle θ between A and B

6) Find the unit vectors T and N that are tangent and normal, respectively, to the curve $y = x^3$ at the point $(2, 8)$

7)Find an equation for the plane through the points $(3, 2, -4)$,$(5, 1, 0)$, and $(-1, 4, 5)$.

8) A shot leaves the shot put thrower's hand 6.5 ft above the ground at a 45^o angle at 62 ft/sec. Where is the shot 2 seconds later? A labeled diagram showing position relative to the shot putter's position at release would be sufficient.

9) Find the length of the curve: $r(t) = ti + t^2j + t^3k$, $0 \leq t \leq 4$. Setup only.

Given $r(t) = ti + t^2 j + t^3 k$

10) Find the unit vector T tangent to the curve when $t = 1$.

Handy Formulas

$|< a_1, a_2, a_3 >| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$< a_1, a_2, a_3 > \bullet < b_1, b_2, b_3 >= a_1b_1 + a_2b_2 + a_3b_3$

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$Proj_B A = \left(\frac{B \bullet A}{A \bullet A}\right) A$

$A \bullet B = |A|\,|B| \cos\theta \qquad \theta = \cos^{-1}\left(\frac{A \bullet B}{|A||B|}\right)$

The plane through (x_1, y_1, z_1) with normal vector $N =< A, B, C >$is
$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

For $r(t) = f(t)i + g(t)j + h(t)k$, the length of the curve traced by r for $a \leq t \leq b$ is $L = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}dt$

$\overrightarrow{T} = \frac{\overrightarrow{v}}{|\overrightarrow{v}|}$

$\overrightarrow{N}$ is a unit vector orthogonal to $\overrightarrow{T}$.